Analyzing Temporal Dynamics in Music:
Differential Calculus, Physics, and Functional Data Analysis Techniques

Bradley W. Vines, Regina L. Nuzzo,
and Daniel J. Levitin
Department of Psychology, McGill University

This article introduces theoretical and analytical tools for research involving musical emotion or musical change. We describe techniques for visualizing and analyzing data drawn from time-varying processes, such as continuous tension judgments, movement tracking, and performance tempo curves. *Functional Data Analysis* tools are demonstrated with real-time judgments of musical tension (a proxy for musical affect) to reveal patterns of tension and resolution in a listener's experience. The derivatives of tension judgment curves are shown to change with cycles of expectation and release in music, indexing the dynamics of musical tension. We explore notions of potential energy and kinetic energy in music and propose that *affective energy* is stored or released in the listener as musical tension increases and decreases. Differential calculus (and related concepts) are introduced as tools for the analysis of temporal dynamics in musical performances, and *phase-plane plots* are described as a means to quantify and to visualize musical change.

Received September 5, 2003, accepted March 28, 2005

Music is said to represent the dynamics of human emotion. Conveying the changing nature of those emotions is fundamental to musical performance (Scherer & Zentner, 2001), and understanding this process is among the most important problems in music cognition (Jones, 1993; Meyer, 1956). Indeed, Aristoxenus (364-304 B.C.E.)—perhaps the first music cognition theorist—argued that music can only be understood by studying both the musician and the *mind* of the listener. Music listening also involves lower-level perceptual processes. The human mind perceives relations between simultaneous elements of sound (e.g., notes in a chord) as well as relations between sequential elements of sound (Bregman, 1990; Dowling & Harwood, 1986; Lerdahl & Jackendoff, 1983). Melodic perception, which develops from infancy (Dowling, 1999; Plantinga & Trainor, in press; Saffran, 2003; Trehub, 2003), relies on a sensitivity to change over time, as does the perception of harmonic chord progressions and rhythmic relations.

In this article, we introduce new analytic techniques drawn from Newtonian physics that are useful for quantifying the dynamic nature of stimuli that change over time, such as music, and human participants' dynamic reactions to these stimuli. Shepard (1984) has persuasively argued for the plausibility of analogies between physics and human cognition with his ideas of resonant kinematics for perceiving and imagining sensory stimuli. The techniques we introduce will allow researchers to solve several problems inherent to dynamic data sets, not the least of which are issues associated with multiple comparisons and repeated measures when the data set (sampled responses to music over real time) may contain thousands of observations. (A subset of these new techniques was first applied to music in McAdams, Vines, Vieillard, Smith, & Reynolds, 2004.)

In what follows, we will refer to time-dependent processes in music as "musical dynamics," based on the terminology introduced by Jones (1993). The effect of temporal context in music—what has played before and what is about to play—continuously influences a listener's experience. An identical physical stimulus may be perceived differently, depending on the context (Jones, 1993; Shepard, 1984); thus music perception is a dynamic, time-dependent process. Changes in loudness in a musical performance are but one example of temporal dynamics in music in the larger Jonesian sense. Other examples include fluctuations in tempo, changes in pitch, and adjustments to timbre.

The study of emotions has recently become a central focus in music cognition research (Juslin & Sloboda, 2001; Scherer, Zentner, & Schacht, 2002). The *continuous tracking* technique, introduced by Nielsen (cited in Madsen & Fredrickson, 1993) and further explored

Music Perception volume 23, issue 2, pp. 137-152, issn 0730-7829, electronic issn 1533-8312

by Madsen, Fredrickson, Krumhansl, and other researchers (Fredrickson, 2000; Krumhansl, 1996, 1997; Krumhansl & Schenck, 1997; Madsen & Fredrickson, 1993; Schubert, 2001; Sloboda & Lehmann, 2001; Vines, Krumhansl, Wanderley, & Levitin, in press) has become an important tool for measuring musical emotion. In an experiment that employs this form of measurement, participants continuously adjust a computer input device to report their ongoing, real-time judgments of the emotional, structural, or aesthetic content in a piece of music. *Functional Data Analysis*[1] tools (FDA; Ramsay & Silverman, 1997, 2002) are ideal for studying continuous measures to reveal underlying structures in the data and relations between the data and other continuous processes, such as tempo change, loudness, rate of movement, and note density (Levitin, Nuzzo, Vines, & Ramsay, 2005). FDA was developed primarily as an alternative to general linear model-based statistics that assume the dependent variables come from independent, discrete observations; FDA treats a curve representing multiple observations as the fundamental unit of analysis. Although continuous data may be obtained in a variety of ways, we will use continuous tracking data in this article as an illustrative and typical example.

This article introduces the interpretation of judgment curves obtained in continuous tracking paradigms as well as the analysis of the *derivatives* of those curves. We demonstrate a way to visualize the data with *phase-plane plots* to reveal the dynamics of change in musical performance. FDA enables researchers to pinpoint in time—with reference to the musical score—when participants perceive important musical and emotional events. We hope to provide new insight into musical processes that involve change, such as tension and release in music, and to show how FDA can be applied to address a broad range of research questions in music perception and cognition.

Tension in Music

Musicians and composers speak metaphorically of tension and release in music as dynamic processes. Musical tension gives rise to an emotional experience in listeners that references real-world, nonmusical counterparts, such as tension in physical objects, in the body, and in social situations. These common life experiences cause a convergence of meaning for "tension" across individuals (Fredrickson, 1997), thus making tension a practical index of emotional experience in scientific research.

The experience of "release" complements the experience of "tension," and the ebb and flow of these opposites elicits emotional responses to music (Krumhansl, 2002; Patel, 2003). Generally, music theorists view tension in a piece of music as being related to phrasing structure (A. Vishio, personal communication, June, 2003), and empirical investigations have found that tension tends to build up during a phrase, to peak toward the phrase ending, and then to subside, often rapidly (Krumhansl, 1996; Krumhansl & Schenck, 1997). Musicians and composers employ tension in a variety of ways by utilizing listeners' expectations and by exploiting basic psychophysical perceptual principles (Balkwill & Thompson, 1999; Meyer, 1956). Fundamental to all uses of tension is its relation to "release"—the inevitable decrease in tension (Storr, 1972).

Measuring Tension in Music: The Continuous Judgment

The continuous tension judgment, introduced by Frede Nielsen (see Madsen & Fredrickson, 1993; Fredrickson, 1995), is a way to assess a participant's real-time experience of a musical piece. Participants indicate the ongoing tension they feel while listening to a musical piece by adjusting a dial-shaped Continuous Response Digital Interface (Fredrickson, 1995, 1997, 1999, 2000; Madsen & Fredrickson, 1993), a foot pedal with a range of motion (Krumhansl & Schenck, 1997), or a moveable slider (McAdams et al., 2004; Vines et al., in press; Vines, Wanderley, Krumhansl, Nuzzo, & Levitin, 2003). The apparatus type does not appear to materially affect the judgment itself or its contour through time. These tension judgments are influenced by a wide variety of structural, harmonic, and rhythmic features in music, including pitch height, note density, loudness, and harmonic dissonance (Fredrickson, 1995; Madsen & Fredrickson, 1993; Krumhansl, 1996; Krumhansl & Schenck, 1997); they are informative about a person's affective state (Fredrickson, 1995) and are robust across participant populations differing in age, level of musical skill, and degree of familiarity with the musical selection being judged (Fredrickson, 1997, 1999, 2000). Tension judgments also correlate with emotional states and with physiological measures (Krumhansl, 1997; Krumhansl & Schenck, 1997). Furthermore, music theoretical analyses of tension (Lerdahl, 1996, 2001) predict judgments of tension in music (Bigand, 2003; Bigand & Parncutt, 1999; Lerdahl & Krumhansl, 2003; Krumhansl, 1996; Patel, 2003; Smith & Cuddy, 2003; Vega, 2003).

---

[1]Functional Data Analysis software tools are available to all interested researchers at http://ego.psych.mcgill.ca/pub/ramsay. The data analysis functions can be downloaded in Matlab code from that Web site. Sample applications of those functions, which provide a template for doing analyses, are also obtainable from the site.

Taken together, there is thus a growing body of evidence for the validity and robustness of the continuous tension judgment as a measure that captures listeners' real-time affective experience of a musical piece.

Interpreting Continuous Judgment Responses: The Meaning of Derivatives

Participants in continuous tension experiments are asked to move some device in one direction as their experience of tension increases and in the other direction as their experience of tension decreases (e.g., up and down on a slider, in and out on spring-loaded tongs). In the remainder of this section, we introduce an interpretation of the first two derivatives of tension curves thus obtained in a musical context. We conceptualize the first derivative as the *affective velocity* of (a person's reactions to) a selection of music, and the second derivative as the *affective acceleration*. The descriptor "affective" is used instead of "tension" for the derivatives because (as discussed above) judgments of tension can serve as a proxy for affective experience or emotion. More specifically, it is likely that tension corresponds to the "arousal" dimension of emotion, as opposed to "valence" (McAdams et al., 2004; Russell, 1979; Schubert, 2001); this claim is supported by Krumhansl and Schenck's (1997) finding that continuous judgments of tension were consistently correlated with continuous judgments of fear, an emotion closely related to arousal. Additionally, the processes we are describing pertain to general emotional experience; thus the term "affect," which is more general than "tension," is suitable.

Schubert (2002) introduced the notion of comparing continuous judgment derivatives in order to determine if two curves are indeed similar. He showed, for example, that Pearson correlations are more reliable when the *derivatives* of the measures are compared, versus the observed curves, noting that "differenced data refers to the change in emotional response which takes place from moment to moment" (p. 230). Our goal is to extend this notion by analyzing two derivatives in an effort to quantify affective aspects of musical experience.

The concepts of affective velocity and acceleration can be illustrated with an example using idealized tension curves. Music contains periodicities at hierarchical levels (Jones, 1993; Krumhansl, 1996; Lerdahl & Jackendoff, 1983; Lerdahl, 2001; Temperley, 2001), and for simplicity we will illustrate this with a sinusoidal wave. Figure 1.1 shows a hypothetical tension curve made in response to a musical piece that induces a cosine wave pattern of experienced tension for listeners. The tension level begins at *a* with an absolute minimum and then increases to a theoretical absolute maximum tension level at *c*, passing through an intermediate level, *b*. A release in tension occurs as the curve passes through point *d*, and the pattern begins again at a'. This idealized sinusoidal model exemplifies a simple, straightforward case in order to illustrate the interpretation of tension curves and their derivatives. The tension and release cycle can, of course, occur at a slow (Figure 1.2) or fast (Figure 1.3) rate.

Derivatives and Differential Calculus

Music is dynamic, and the calculus (Leibnitz, 1686; Newton, 1726) was invented to study changes across time. If Y represents our original function (the tension curve associated with a musical piece), then the first derivative (Y') represents the instantaneous rate of

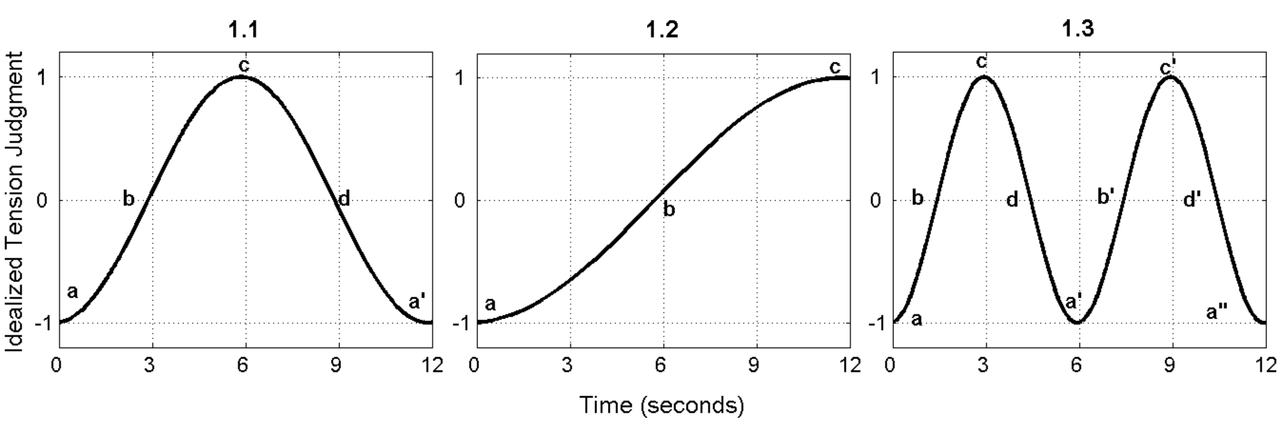

FIG. 1. (.1) A sinusoidal wave representation, with shifted phase, as an idealized model for musical tension judgments. (.2) A cosine wave with half the frequency of the wave in (.1), modeling tension judgments for a musical piece with slower emotional changes. (.3) A cosine wave with twice the frequency of the wave in (.1), modeling tension judgments for a musical piece with rapid emotional changes.

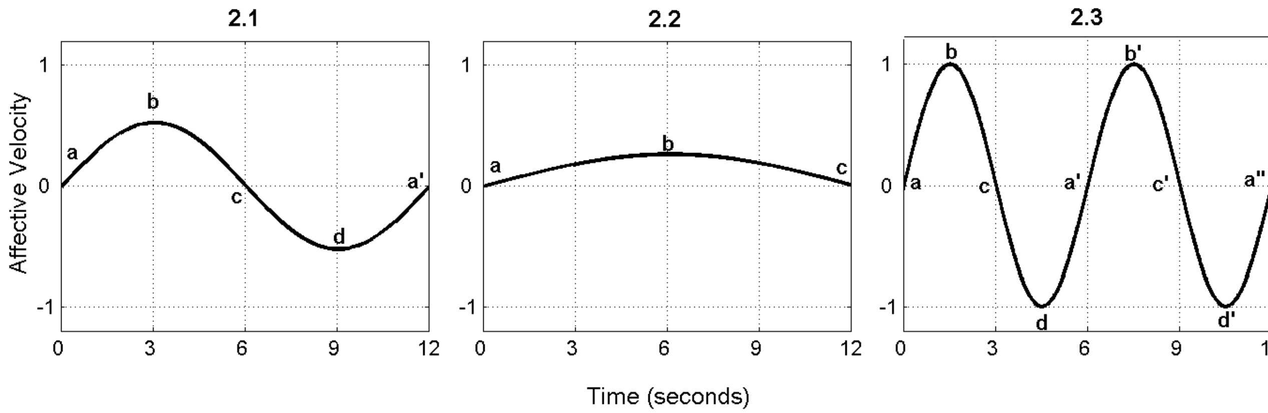

FIG. 2. The first derivatives of the idealized tension curves shown in Figures 1.1 through 1.3.

change of our primary measure, and the second derivative (Y'') represents the instantaneous rate of change of the first derivative. It is certainly mathematically possible to take the first and second derivatives of tension curves, but do these derivatives have musical (or music-cognitive) meaning? We explore this question in the data analysis described below.

Affective velocity, the *rate of change* of a tension curve (Y'), reveals how quickly the experience of tension is increasing or decreasing for a listener. The first derivative of the idealized tension curve of Figure 1.1 is shown in Figure 2.1.

The derivative curve reveals a combination of the composer's and performer's manipulation of tension as well as the emotional dynamics of a listener's experience by showing how fast tension changes. From Y', we can learn not only that tension is increasing, for example, but also whether the tension is increasing quickly or slowly. Note for example the difference between Figures 1.1, 1.2, and 1.3 and the corresponding derivatives in Figures 2.1, 2.2, and 2.3. In Figure 1.2, the cosine wave has a period that is twice as long as the curve in 1.1; the tension and release are spread out over a longer period of time, yielding a first derivative of smaller magnitude (Figure 2.2). Such a pattern would be likely for a musical piece with a relatively slow progression (e.g., "My Funny Valentine" played by Miles Davis, or Ravel's Bolero). The power of this analytical approach lies in the ability to *quantify* musical tension over time. In Figure 1.3, the cosine wave has a period that is half as long as the curve in Figure 1.1; the corresponding first derivative, indexing rate of change, has a greater magnitude (Figure 2.3). Such a pattern, characterized by rapid change, might be found for a musical piece with rapid fluctuations in tension (e.g., "Cherokee" played by Charlie Parker). As described here, the first derivative does have musical meaning: the rate of change of a person's emotional response to music (Schubert, 2002). One could imagine categorizing different musical pieces by the magnitude of their first derivative in emotional response.

Whether or not the derivatives of affective experience, velocity and acceleration, have a distinct perceptual reality (i.e., that the experience of acceleration is different from the experience of a steady change in tension) is an empirical question that must be approached through experimental investigation. In this article, we hypothesize that affective acceleration and affective velocity are related to unique dimensions of the musical experience.

Energy Transfer in Musical Dynamics

Given that tension increases and decreases throughout a musical piece, we can introduce an analogy in which fluctuations in tension correlate with an ebb and flow of affective (or musical) energy that is stored up and released within a listener. Increasing tension in the piece generates a buildup of *affective energy* in the listener; a subsequent decrease in tension results in a release of the pent-up energy. (In this way, the listener functions like a capacitor by storing up a charge until a threshold is reached, at which point the charge is released.) The storage and release of affective energy causes changes to a listener's experience of emotional content in a piece and is presumably under the control, to a large extent, of the composer and performers. That is, stored musical energy has the potential to alter the emotional state of a listener when it is ultimately released.

Musical experience (and performance art in general, such as dance and theater) usually entails a sequence of energy *transfers* over time; the emotive experience of a performance increases from a baseline level, builds up to a heightened level, drops to a state of release, and then builds up again (see Krumhansl, 2002; Meyer, 1956). Krumhansl and Schenck (1997) found that emotional responses to both dance and music built up over the duration of phrases and ultimately tended to decrease toward phrase endings. McAdams and colleagues (2004) found similar patterns of peaks and decays in their participants' emotional responses to a contemporary electroacoustic musical piece. In this sense, musical performance can be thought of as involving the transfer of energy from the composer and performers to the listener, and a cyclical process of storage and release of that energy within the listener.

The rate at which *affective velocity* (the first derivative) changes over time is expressed by the second derivative, which we refer to as *affective acceleration*. Having established that the composer is leading the listener toward increasing tension (for example), the second derivative, affective acceleration, reveals whether the listener is being brought to that point of tension at a constant rate, at an increasing rate, or at a decreasing rate. When the rate of change is increasing, it may feel as though a goal is being reached more and more quickly, as in the following thought example (B. Thompson, personal communication, August 20, 2004). Imagine a horror movie in which a character is slowly descending a staircase into a dark basement—the tension has a positive velocity. Suddenly, the villain, lurking in wait in the darkness below, becomes visible to the audience—the rate of change in tension increases, and an increasing slope of the tension curve is evidence of positive affective acceleration.

POTENTIAL AND KINETIC ENERGY

We are borrowing the concepts of potential and kinetic energy from Newtonian physics to describe the dynamics of affective change in music and to better understand how derivatives of tension curves relate to musical meaning. Past research has explored analogies between musical phenomena and the laws of physics by comparing dynamic temporal processes in music, such as ritardando, with processes in the physical world, such as the movement of objects through a gravitational field or a runner's rate of deceleration (Clarke, 1999; Davidson & Correia, 2002; Friberg & Sundberg, 1999; Todd, 1999). These studies have revealed that aspects of change in music are structured in a similar way to movement and change in the physical world. Larson (2004) drew parallels between melodic expectation and the forces of inertia, gravity, and magnetism, arguing "we experience musical motions metaphorically in terms of our experience of physical motions" (Larson, 2004, p. 462). Shepard (1987, 1995) posits that the human mind evolved to incorporate external physical laws into mental representation, explicitly mentioning the principles of physical dynamics and movement (analogous to our temporal derivatives) among those universal constraints that shaped human cognition (Shepard, 1984). The analytical tools that have emerged in physics to describe flux in the physical world are also useful for understanding musical fluctuation and experience.

In our idealized example, we can say that affective experience has a tendency to gravitate toward a stable state of complete release. This is the baseline ($y = -1$) seen in Figure 1. Musical pieces may not always resolve to a point of low tension, especially in twentieth-century art music, as the composer flouts various maxims of musical expectation or seeks different aesthetic aims than a sense of completion or release. (This is perhaps the musical equivalent of flouting Grice's conversational maxims in language; Grice, 1975.) However, this does not negate the value of derivatives in understanding the ongoing nature of changes in tension throughout a piece—in fact, the derivative analysis we are proposing does not depend on how a piece ends, but rather is a tool for analyzing the changes in musical tension throughout; it can, furthermore, permit us to quantify the state of tension/resolution that exists at the end of a piece.

In physics (and in our analogy to music), *potential energy* is synonymous with *stored energy*; here, stored musical energy is the potential to alter the emotional state of a listener. The potential for change is at its greatest at a point of maximum tension in music, such as point *c* in Figure 1. The music has brought listeners to a peak of tension; hence, a resolution is likely to follow to satisfy the listeners' expectations and to adhere to musical convention. (There may also be physiological constraints that prevent a person from maintaining a high-tension experience for very long durations of time.) The higher the tension becomes—or the longer that it is held at a high level—the stronger the tendency or "pull" exerted by the music to return the listener to a more moderate state; in our idealized tension curve, this pull is indicated by an acceleration in the direction of the pull—it is negative at point *c* of Figures 1 and 2, for instance. (The slope of affective velocity is negative at point *c*; see Figure 2.) Note that in our simplified model, which represents the pattern of musical tension as a sinusoid, nonzero values for velocity and acceleration always co-occur, except at the maxima and minima; thus we can use the existence of acceleration as an indicator that potential energy is being created. But real musical pieces could certainly have stretches of positive (or negative) velocity while the acceleration is zero.

A strong potential for change in emotional states also occurs when the tension level is very low—small manipulations by performers and composers can easily increase tension, and listeners may expect such an increase as well. At such points (*a*, and *a′* in Figures 1 and 2), there will be high potential energy for increasing tension levels, as indicated by a strong positive acceleration in the idealized tension curves. (The slope of affective velocity, shown in Figure 2, is positive at points *a*, and *a′*.) Therefore, the maxima in potential energy occur at both points *a* and *c* of Figures 1 and 2.

In addition, we hypothesize that kinetic energy, the energy of movement, is related to the rate of change in emotion—affective velocity. If musical energy is conserved over time, as in our idealized tension model, then as a listener's kinetic energy increases, potential energy decreases—the transfer of musical energy facilitates a change in emotional state. This exchange of energy continues until potential energy and the associated affective acceleration decrease to zero, at which point kinetic energy reaches a maximum (points *b* and *d* in Figure 1). The potential energy for change in the opposite direction then increases, while kinetic energy and the associated affective velocity decrease toward zero at points *a* and *c*. This trade-off between kinetic energy and potential energy, velocity and acceleration, continues throughout the hypothetical tension judgment. In real music, the composer or performers contribute energy to the music to create contours of experience that are not strictly periodic.

Phase-Plane Plots

A Functional Data Analysis technique for visually representing the dynamics of continuous processes is the *phase-plane plot*, in which the second derivative is plotted against the first derivative (Ramsay & Silverman, 2002). In the musical context we have introduced, the phase-plane plot graphs affective acceleration against affective velocity. Figure 3.1 shows the phase-plane plot for the idealized cosine curve of Figure 1.1 (it is thus a plot of Figure 2.1 versus its derivative). Purely oscillatory behavior yields a perfectly circular phase-plane plot. A larger radius in the plot corresponds to a greater amount of musical energy transfer, as would occur for a piece of music with rapid changes in tension or in the velocity of tension. A piece of music with no changes in tension would yield a single point at the center. Figure 3.2 shows a plot in which there are large accelerations and relatively small velocities, and Figure 3.3 shows a plot in which there are small accelerations and large velocities. Note that the phase-plane plot does not explicitly plot time, though time markers along the path can be annotated, as we will demonstrate with Figure 6.

We can relate the poles of the phase-plane plot to the point markers in Figures 1.1 and 2.1. Points of

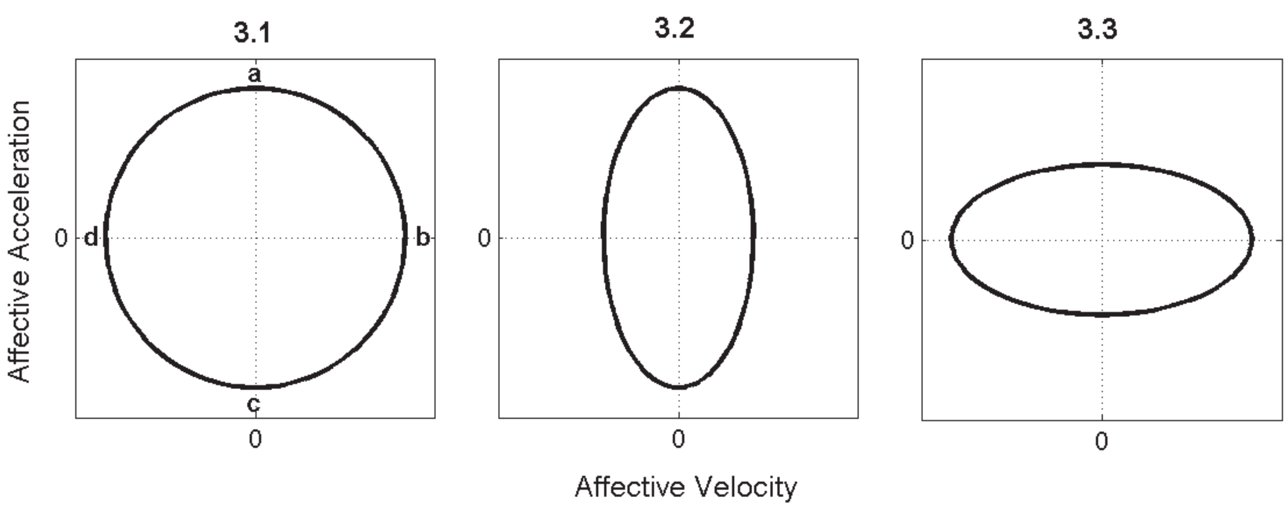

FIG. 3. (.1) Phase-plane plot for the cosine wave shown in Figure 1.1, with affective acceleration plotted against affective velocity. (.2) A hypothetical phase-plane plot for a judgment with large affective accelerations and small affective velocities. (.3) A hypothetical phase-plane plot for a judgment with small affective accelerations and large affective velocities.

maximum kinetic energy and affective velocity occur at *b* and *d*, as indicated by their distance from the y-axis in Figure 3. The maximum positive velocity occurs at point *b*, the maximum negative velocity occurs at point *d*, and the velocity is zero at the center of the plot (the origin). Points *a* and *c* are points of maximum potential energy, as indicated by their distance from the x-axis in Figure 3. Point *a* has maximum positive acceleration and point *c* has maximum negative acceleration. The acceleration is 0 at the center of the plot. At *a* and *c*, the first derivative (affective velocity) is 0.

An Example With Real Data

To illustrate the new application of techniques based on differential calculus and phase-plane plots, we will discuss a segment of the data collected in a recent study by Vines et al. (in press).

Data Collection

Thirty musically trained participants were randomly divided into three treatment groups and were presented with a performance of Stravinsky's second piece for solo clarinet (1920/1993). (Final $N = 28$ after discarding two outliers.) One group (auditory only, $n = 9$) heard the performance without seeing it, a second group (visual only, $n = 9$) saw the performance without hearing it, and the third (auditory+visual, $n = 10$) both saw and heard the performance. This piece was chosen because it is an unaccompanied work that lacks a metric pulse; hence the performer was free to move expressively and idiosyncratically, and the experimental participants who saw the performance could not rely on metrically based body movements to make their judgments.

Each participant performed the continuous tension judgment using a slider with a span of 7 cm, sampled at 10 Hz. This rate is relatively high compared to past studies involving continuous musical ratings. Krumhansl and Schenck (1997) used 4 Hz, as did Krumhansl (1996), and Madsen and Fredrickson (1993) used 2 Hz, for example. Madsen and Fredrickson (1993) found that sampling rates above 2 Hz did not add information to the tension curves that they obtained. If 2 Hz is close to the maximum frequency of relevant information, then a 10 Hz sampling rate more than adequately satisfies the Nyquist theorem, which states that the sampling rate must be at least twice the frequency of the signal being measured; thus no distortions due to sampling rate were introduced. In our data, the average change per sample was approximately 1% of the total range of the slider. The following instructions were given:

> Use the full range of the slider to express the TENSION you experience in the performance. Move the slider upward as the tension increases and downward as the tension decreases. Begin with the slider all the way down.

Each participant used his or her best intuition about the meaning of "tension," following previous studies (Fredrickson, 1995, 1997, 1999, 2000; Krumhansl, 1996, 1997; Krumhansl & Schenck, 1997; Madsen & Fredrickson, 1993).

Analysis

DATA PREPARATION, SCALING, CENTERING

The raw data consisted of 28 records (one for each participant) of 800 samples each (80 seconds of music at 10 Hz sampling rate). The values obtained from the MIDI slider ranged from 0 to 127. To begin the analysis, each participant's judgment was *scaled* to an interval of 0 to 1. Due to the scaling, each sample over time represented the proportion of the maximum tension experienced by a participant during the entirety of the piece. (Other transformation schemes and interpretations are discussed in Vines et al., in press). Each vector then was *centered* by subtracting the median of all values from each element in the vector (a process also referred to as "zero-meaning"). This, in essence, equated the central tendency in each participant's judgment. The scaled and centered data for a single representative participant in the auditory-only condition are shown in the top panel of Figure 4.

CREATING A FUNCTIONAL OBJECT

In order to calculate the derivatives of the responses, we converted the data from discrete points (at 10 Hz) into smoothed representations known as *functional objects* (Levitin et al., 2005; Ramsay, 2000; Ramsay & Silverman, 1997). Ideally, such functional curves eliminate noise and collection artifacts in the discrete, raw data, and they approach the true, underlying process from which the data were collected with greater accuracy.

Each observed record of discrete data was modeled by a basis expansion of 150 sixth-order B-splines (Deboor, 1978; Unser, 1999) and a lambda smoothing parameter of 0.1, using custom software (Ramsay, 2003) written in Matlab (The Mathworks Inc., 2003). See Levitin and colleagues (2005) for a description of Functional Data Analysis modeling and guidelines for choosing the number and order of B-splines for

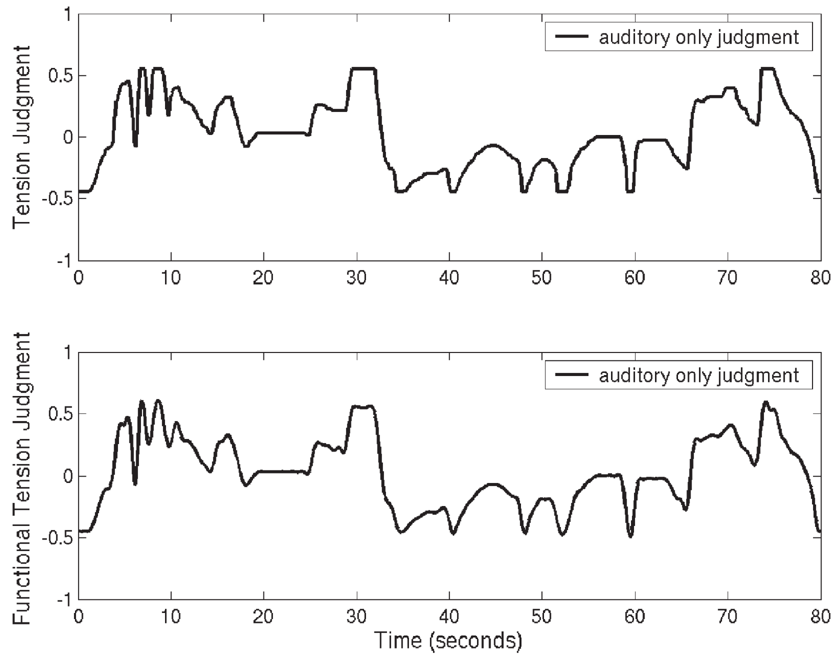

FIG. 4. (Top panel) Scaled raw data obtained from a single representative participant in the auditory-only condition of the experiment described in text. The measurement was a continuous judgment of musical tension. (Bottom panel) The same judgment after functional modeling. Note the similarity between the two panels, indicating an accurate B-spline estimation.

smoothing. The bottom panel of Figure 4 shows the FDA-modeled data for the auditory-only participant whose raw data appear in the top panel. The visual similarity between the two panels of Figure 4 demonstrates the accuracy of the B-spline approximation.

INTERPRETATION OF THE DATA

In the empirical data we are referring to, we assume that the position of the slider corresponds to each participant's real-time experience of the tension in the musical piece, perhaps with some response delay or anticipation. Past research with continuous measures found response delays from 1 second (Kuwano & Namba, 1985) to over 3.25 seconds (see Schubert, 2001; Smith & Cuddy, 2003). Here, we are not concerned with the response delays, which are relatively minor compared to the time spans of interest, and methods exist for normalizing delays among participants (Levitin et al., 2005; Ramsay & Silverman, 1997). The individual participants vary around a mean response lag. Mathematically, this delay has the effect of adding a small constant, which does not materially affect interpretations at the level of musical phrases.

PHASE-PLANE ANALYSIS WITH DATA

To illustrate the preceding concepts, we consider Vines et al. (in press) study introduced earlier, using the methods presented in this article. Figure 5.1 shows the mean functional tension judgment for participants in the auditory-only condition. All 80 seconds of the judgment mean are shown. There are clear periodicities in the mean curve that correspond to the musical structure of the piece. The composition has three main sections (Friedland, n.d.; A. Vishio, personal communication, April, 4, 2003) that differ in musical content. The first and third sections (0 to ~33.5 seconds and ~66 seconds to the end, respectively) mirror each other musically in that they both consist of fast-moving lines with a wide pitch range that elaborate upon similar thematic material. The second section (~33.5 to ~66 seconds) consists of slower-moving lines at a low pitch height that elaborate upon different thematic material. The mean tension curve in Figure 5.1 is relatively high in section one, with some small-scale fluctuations. The second section is marked by a low tension rating with two major subsection peaks. The tension level returns to a high level in the third section, which shows another subdivision. Overall, the global shape follows a sinusoidal path with two full cycles (low to begin, high in section one, low in section two, high in section three, and low to end) with small-scale cycles of tension and release adding fluctuation within the global sinusoid. First and second derivatives of the tension mean are depicted in Figures 5.2 and 5.3, respectively.

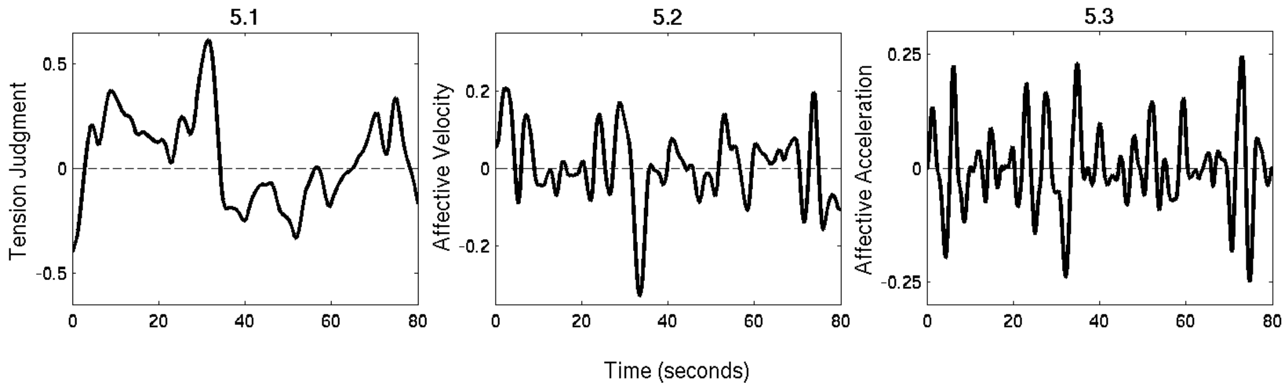

FIG. 5. (.1) The smoothed mean curve obtained from nine functionalized auditory-only judgments. (.2) Representing affective velocity with the first derivative of the mean curve. (.3) Representing affective acceleration with the second derivative of the mean curve.

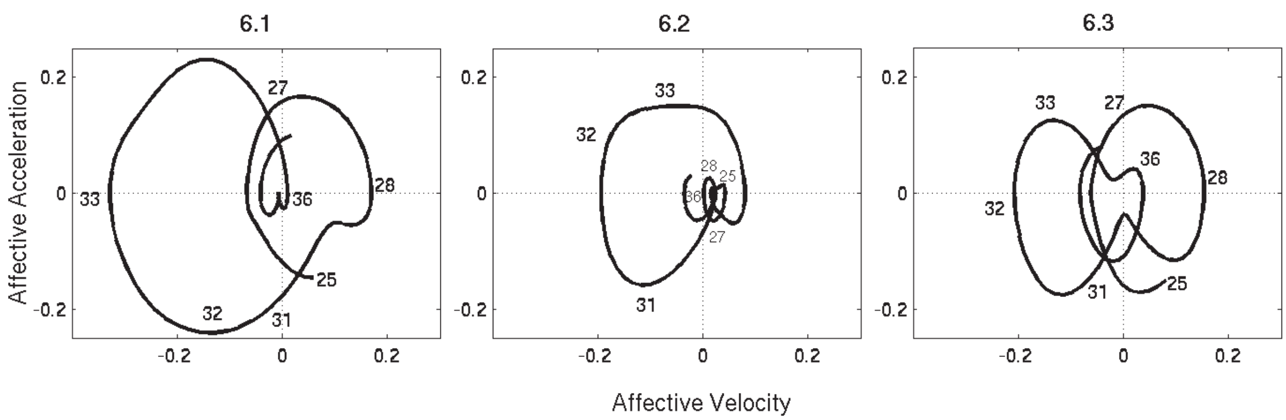

FIG. 6. (.1) Phase-plane plot for the auditory-only tension mean, showing Y' vs. Y'', or *affective velocity* vs. *affective acceleration*. Arabic numerals in the graph mark clock time in seconds from the beginning of the piece. (.2) Phase-plane plot for the visual-only mean judgment. (.3) Phase-plane plot for the auditory+visual mean judgment.

The phase-plane plot in Figure 6.1 plots a segment of the curve in 5.3 against the corresponding segment of the curve in 5.2, to illustrate three things: (a) the dynamics of tension experienced in a performance of Stravinsky's piece, (b) the cycles of energy transfer between potential and kinetic, and (c) the ongoing changing relations between affective velocity and affective acceleration in the piece. For brevity, we have plotted only the judgments made during an especially interesting 15-second section of the piece: 25 seconds from the beginning up to 40 seconds from the beginning. (Arabic numerals indicate the time markers in clock seconds.) This segment corresponds to an important transition in the musical piece when the first section ends with a high peak note and the second section begins after a rest; the musician's body movements continued through the silence. The time markers correspond to specific events in the performance.

Figure 7 (please see color plate) shows an image of the clarinetist at each time marker as well as a pointer to the corresponding location in the musical score. Graphs of the mean tension curves, the first derivative curves corresponding to affective velocity, and the second derivative curves corresponding to affective acceleration are shown below the score for all three presentation groups (auditory only, visual only, and auditory+visual) over a duration that includes all of the important time points. The phrase that builds up to a high peak note begins in second 27. In second 28 the high peak note is reached and held. In second 31, the high note ends and a silent rest ensues, with the performer's movements continuing into the silence. The performer begins to move in preparation for the next section in second 32; he takes a breath and moves the clarinet bell downward in anticipation of the new phrase, which starts in second 33. The second phrase in the new section begins in second 36.

The curve in Figure 6.1 begins with moderate levels of energy change, as indicated by its proximity to the origin at second 25. The energy change reaches greater levels from second 31 through second 33, before returning to a low energy state at second 36. The rounded path that circles the origin indicates that the listeners reported a varied and dynamic (i.e., rapidly changing) musical experience during this section of the piece; both positive and negative affective velocities were felt as well as positive and negative affective accelerations. An analysis of the relation between acceleration, velocity, and the performance stimuli yields insight into listeners' emotions and demonstrates the utility of phase-plane plotting.

The phase-plane plot shown in 6.1 reveals emotional dynamics that were driven by events in the performance. Affective velocity becomes positive in second 27 (acceleration was already positive) just as the phrase that built to the peak note began. When the high peak note is reached in second 28, the velocity achieves a positive maximum and the acceleration passes through 0, becoming negative. The change in acceleration signals the end of an emotional buildup. Once the target peak note is reached, acceleration becomes negative, signaling a pull in potential energy toward release. This moment in the energy transfer is analogous to point *b* in our idealized model (compare point *b* in Figure 3.1 to second 28 in Figure 6.1). When the high note ends in second 31, the velocity becomes negative. The auditory-only group could not see the musician; hence his movements in second 32 could not have influenced the emotional dynamics experienced by the group. Just as the new musical phrase begins in second 33, the affective acceleration becomes positive. This moment in energy transfer is analogous to point *d* in the idealized tension curve. In second 33, an event in the music (the onset of a new phrase) appears to have directly influenced affective acceleration, which changed from negative to positive. Though the judgment of tension continues to change with negative velocity, the acceleration grows more and more positive in response to the musical change. This example demonstrates the utility of phase-plane plotting: Hidden dynamics in the dependent variable were revealed. We posit that changes in music can act on a listener's experience at the level of affective acceleration and that there may be very little temporal delay for such an effect. As the new section continues into second 36, the overall amount of energy transfer dies down, as shown by a phase-plane trajectory that moves closer and closer to the origin.

The phase-plane plot makes it possible to compare the dynamics of two or more judgment curves (e.g., from different participants, different musical pieces, or different conditions of an experiment). In the experiment under consideration here, recall that there were three participant conditions that differed in sensory modality of presentation: auditory only, visual only, and auditory+visual. Figure 6.2 depicts the phase-plane plot for the visual-only condition. We can deduce that the magnitude of energy transfer was relatively weak for the visual-only condition (as indicated by a path that is generally closer to the origin). Note that the difference in phase-plane magnitude involves the derivatives of the mean tension judgment and not the undifferentiated place data, which were scaled to eliminate varying uses of the slider. The visual-only phase-plane plot follows the same general trajectory as the auditory-only phase-plane plot, but with a phase that is shifted forward in time. The graph in 6.2 shows a maximum negative affective acceleration in second 31, whereas the corresponding maximum in 6.1 occurs at least 1 second later in second 32. Additionally, the visual-only phase-plane plot achieves a maximum negative velocity before second 32, which is well before the auditory-only plot reaches its corresponding maximum in second 33. A comparison of the auditory and visual phase-plane plots suggests that during this segment similar emotional dynamics were conveyed by the sound and by the sight of the performance, though the overall magnitude of energy transfer was less in the visual-only condition and emotional information was conveyed earlier by the visual modality.

The first portion of the auditory+visual phase-plane plot shown in Figure 6.3 is nearly identical to the auditory phase-plane plot. The velocity becomes positive in second 27, as the phrase leading to the peak high note begins. When the peak high note arrives in second 28, the velocity reaches a positive maximum and the acceleration becomes negative. The high note ends at second 31, just before acceleration reaches a negative maximum. At this point, the timing of the auditory+visual phase-plane plot deviates from that of the auditory-only plot. The visual content can account for this divergence.

As mentioned above, the clarinetist begins his movements in anticipation of the new section. In second 32, he takes a breath and prepares his clarinet by moving it forward before initiating any sound. Based on the phase-plane plot depicted in Figure 6.3, we posit that the performer's expressive movements influenced participants in the auditory+visual group. During second 32, the acceleration becomes positive; the velocity remains negative, but the rate of change begins to increase once the player's anticipatory movements begin. This is another example for which changes in the performance are reflected as changes in affective acceleration. The auditory-only phase-plane plot does not achieve a positive acceleration until second 33, when the sound begins; those participants could not see the performer and, therefore, could not be influenced by his movements. The fact that the change in acceleration begins earlier for the auditory+visual group suggests that the performer's anticipatory movements (his breathing, body lunging, and swooping down of the clarinet bell) influenced the judgments by providing affective information in advance of the sound.

Summary

The phase-plane plot analysis just described generated insight into the dynamics of emotional experience in music. Visualizing the derivatives of tension judgments revealed patterns of energy exchange that were not obvious in the raw position data. The phase-plane plot enabled comparisons of energy transfer across experimental conditions and brought to light different aspects of musical influence on emotion. Based on this analysis we posit that changes in a musical stimulus can have a direct effect on affective acceleration and that this effect is revealed with a shorter time lag than position data (i.e., a peak in slider position lags well behind the causal music event that influences affective acceleration more quickly). Using a comparison of phase-plane plots across experimental conditions, we also found that a musician's movements influence perception at key points in the music for those who can see the performance. This influence itself is dynamic and evolving over time, making techniques such as the phase-plane plot ideal for analyses.

A striking conclusion of Vines et al. (in press) was that the body movements of musicians do carry meaningful information about the music and that visual information can indeed contribute to the overall experience of emotion and the perception of phrasing structure in a musical performance. The independent contribution of the present article is the finding that emotional dynamics conveyed visually were similar to, but out of phase with, emotional dynamics conveyed aurally during a performance segment; participants who could both hear and see the performances were influenced by the phase-advanced affective information in the visual channel.

Discussion

This article introduces new techniques for theoretical and quantitative analysis of *musical dynamics* (change over time in music). We show that concepts from *differential calculus* are well suited for use in the domain of music cognition and that *physics* analogies can deepen interpretations of emotional response. *Functional Data Analysis* techniques for modeling, smoothing, and taking derivatives are described, and we present *phase-plane plots* as a practical tool for visualizing musical dynamics. We demonstrate the value of the analytical tools that we introduce by means of an idealized model for musical tension and an analysis of continuous tension ratings made in response to a musical performance. Here, tension is treated as a proxy for emotion, based on experimental evidence; hence the ideas discussed may be generalized to affective experience in general.

A primary contribution of this article is to introduce differential calculus as a means for analyzing the dynamics of musical performances. Calculus has refined techniques for exploring rates of change and movement over time. A person's emotional experience is changing continuously while listening to music, and a piece of music that creates strong affective changes can be called "moving." With respect to one's continuous magnitude of emotional experience, we label the first derivative *affective velocity* and the second derivative *affective acceleration* and we show how these derivatives relate to expectation and release, a primary catalyst for affective energy in music.

Terms from physics such as *energy* have an intuitive sense in the domain of music. We suggest here that analogies between physical mechanics and musical dynamics may lead to insights related to emotional change in a listener's experience. In Newtonian mechanics, acceleration (the second derivative of position—*affective acceleration* in the sinusoidal model of musical tension) is related to potential energy while the velocity (the first derivative of position—*affective velocity* in the sinusoidal model of musical tension) is related to kinetic energy. We explicate the links between these concepts and *musical dynamics*, by describing the *transfer of energy* that occurs during a musical piece. Specifically, *affective energy* is stored or released in the listener as tension increases and decreases, and the changing values of the first and second derivatives of tension ratings give evidence of this energy transfer.

This article also introduces *phase-plane plots* as a practical tool for visualizing relations between a measure's first and second derivatives, two latent variables that quantify musical change and affective dynamics in a piece. With this plotting technique it is possible to compare the underlying dynamics of different data sets and to attribute meaning to the velocity and acceleration of a musical measurement. A phase-plane plot analysis of collected tension judgments revealed that the timing of peaks in emotional dynamics differed across presentation conditions. The experience for those who could see the performer was phase shifted in advance compared to those who could only hear, thus demonstrating the influence of the musician's movements through a direct comparison of emotional dynamics across presentation conditions.

The techniques described herein will allow emotion and music cognition researchers to refine their analysis of musical dynamics. Music listening is the experience created by mentally organizing sound over time; therefore precise methods are necessary for characterizing not just musical events with respect to time but also the ways in which those events change—and the ways in which those *changes* change over time. Methods from calculus, physics, and FDA are well suited for applications with musical data, and they have the potential to reveal latent relations in data sets and to enhance our understanding of temporal dynamics in music.

Extensions

Having already introduced the concepts of *energy*, *velocity*, and *acceleration* to a musical context, it is useful to contemplate additional analogies between musical and physical dynamics. Concepts from physics that have meaning in the physical world may well have analogous forms in music (see Davidson & Correia, 2002). Some such concepts already have common currency in music criticism—people speak intuitively about the musical *momentum* of a piece, for example. Whether these can be described by mathematical equations drawn from the physics paradigm is a question for further inquiry. Less intuitive, perhaps, is the concept of *external force*. In physics, we know that an external force must be applied to alter the direction or speed of a physical object in motion. In music, we might hypothesize that the composers and performers apply the external force that influences the direction or speed of affective experience in a listener. Additional physical concepts may be useful for quantifying musical meaning: *friction*, *mass*, *gravity*, *power*, *work*, and *oscillating systems*.

Of course, the limitations of analogies between musical phenomena and physics must be kept in mind. Concepts in physics describe relations and changes in matter and its alternate form, energy. Science has created methods by which to quantify and, to some extent, to control matter and energy. However, no such method is yet available for quantifying musical experience itself, though introspective feedback and physiological measurements can be used as indicators. Tools developed in physics comprise an analytical technology that is suited for processes involving change; musical experience is undoubtedly characterized by change and, therefore, its analysis benefits from the methods developed in physics.

Other Data Sets

We illustrated the methods introduced here with ratings of musical affect. The methods are quite general, however, and they can be applied to any data for which the underlying process is continuous—even if the actual data are "discrete." The data need not be derived from participant judgments either. As an example, Repp (1992, 1996, 1997) has extensively investigated the timing profiles of expressive piano performances. With the methods described here, which were not available when Repp did his seminal work, it would be possible to examine the functional dynamics of tempo changes (such analyses have not yet been performed). For instance, a first derivative taken from the tempo profile of a Chopin Étude would represent the tempo velocity, or the speed at which tempo was changing, and the second derivative would reveal the acceleration of tempo. The derivatives could be used to ask questions such as: How smoothly or monotonically does the performer slow down or speed up? Are there particular points in time when the tempo acceleration is greatest? Does the performer replicate similar tempo profiles, at the level of velocity and acceleration, across performances of the same piece?

Repp extensively characterized interperformer differences by factor analyzing the raw timing (using Inter Onset Intervals) and loudness performance curves. Repp (1992), for example, extracted four principal components to distinguish the interperformer differences among 28 pianists. Functional principal component analysis techniques (fPCA) now exist which differ from the methods used by Repp in that they enable a researcher to explore major modes of variation over time. For example, fPCA might reveal that, compared to the average, one group of performers tends to decrease in tempo at the beginning of a phrase while increasing in tempo toward the end. Functional techniques also yield greater control over the smoothness and visual presentation of the principal component curves. But perhaps the biggest advantage of applying these methods to a data set like Repp's is the ability to analyze *derivatives* of the curves. Functional factor analyses on the derivatives—tempo velocity and tempo acceleration, in this case—provide a new way to describe interperformer variability. These methods may well uncover latent aspects of pianists' styles that will help to further quantify and characterize the performers' aesthetics.

Furthermore, one could correlate the tempo velocity and tempo acceleration with participants' judgments of emotion in the piece to better understand how tempo dynamics contribute to affective experience (Sloboda & Lehmann, 2001). Such an analysis would reveal the functional relations between the two dependent variables, tempo and tension, and their derivatives. Alternate measurements of emotion, such as those obtained by Clynes' sentographic recordings (see Clynes & Nettheim, 1982), are analyzable with the techniques presented here. It would also be possible to apply these methods to the analysis of human movement data, and in particular to the movements of musicians (Wanderley, 2002).

Inferential Statistics

It is possible to examine the sampling variability of functional curves and their derivatives using methods that are still under development, including functional tests of significance (Ramsay & Silverman, 1997), functional bootstrapping (Efron, 1979; Efron & Tibshirani, 1993; Ramsay & Silverman, 2002), and random fields (Shafie, Sigal, Siegmund, & Worsley, 2003). These methods allow researchers to determine whether a derivative differs significantly from zero or whether two sets of curves differ significantly from one another.

Functional Data Analysis and Time-Series Analysis

FDA and Time-Series Analysis (TSA) are complementary areas of statistics, each of which is tailored to a different aspect of data sampled over time. TSA focuses on short-term stochastic behavior in a system that satisfies the assumptions of stationarity; that is, TSA assumes that the patterns in data will remain constant over time with a relatively unchanging mean and an unchanging variance. FDA is designed to explore global changes in data over time, including changes in mean and changes in variance within and between curves. FDA does not maintain assumptions about stationarity. Additionally, Functional Data Analysis offers analogous tools to those used in traditional statistics (e.g., correlations, general linear modeling, principal components analysis, analysis of covariance, and significance testing) but with functions of time as the units for analysis as opposed to data points. FDA and TSA techniques may be used in coordination, for example, to analyze the stochastic properties in a time series, using TSA, after the global patterns of change and variation have been identified and removed, using FDA (J. Ramsay, personal communication, September 13, 2004).

Authors' Note

A version of this article was submitted in partial fulfilment of the requirements for the PhD degree by the first author at McGill University. The authors are grateful to Jim Ramsay, Bruno Repp, Associate Editor Bill Thompson, and the anonymous reviewers for helpful comments on earlier drafts of this manuscript. This work was supported by grants from NSERC, CFI, SSHRC, and VRQ to Daniel J. Levitin, and a J. W. McConnell McGill Majors Fellowship and a CIRMMT Doctoral Fellowship to Bradley W. Vines.

Address correspondence to: Daniel J. Levitin, Department of Psychology, McGill University, 1205 Avenue Penfield, Montreal, QC H3A 1B1 Canada. E-MAIL *dlevitin@psych.mcgill.ca*

References

BALKWILL, L. L., & THOMPSON, W. F. (1999). A cross-cultural investigation of the perception of emotion in music: Psychophysical and cultural cues. *Music Perception, 17*, 43-64.

BIGAND, E. (2003). Traveling through Lerdahl's tonal pitch space theory: A psychological perspective. *Musicae Scientiae, 7*, 121-140.

BIGAND, E., & PARNCUTT, R. (1999). Perceiving musical tension in long chord sequences. *Psychological Research, 62*, 237-254.

BREGMAN, A. S. (1990). *Auditory scene analysis: The perceptual organization of sound*. Cambridge, MA: MIT Press.

CLARKE, E. F. (1999). Rhythm and timing in music. In D. Deutsch (Ed.), *The psychology of music* (2nd ed., pp. 473-500). San Diego: Academic Press.

CLYNES, M., & NETTHEIM, N. (1982). The living quality of music: Neurobiologic patterns of communicating feeling. In M. Clynes (Ed.), *Music, mind, and brain: The neuropsychology of music* (pp. 47-82). New York: Plenum Press.

DAVIDSON, J. W., & CORREIA, J. S. (2002). Body movement. In R. Parncutt & G. E. McPherson (Eds.), *The science and psychology of music performance: Creative strategies for teaching and learning* (pp. 237-250). Oxford: Oxford University Press.

DEBOOR, C. (1978). *A practical guide to splines*. Berlin: Springer-Verlag.

DOWLING, W. J. (1999). The development of music perception and cognition. In D. Deutsch (Ed.), *The psychology of music* (2nd ed., pp. 603-625). San Diego: Academic Press.

DOWLING, W. J., & HARWOOD, D. L. (1986). *Music cognition*. Orlando, FL: Academic Press.

EFRON, B. (1979). Bootstrap methods: Another look at the jackknife. *Annals of Statistics, 7*, 1-26.

EFRON, B., & TIBSHIRANI, R. J. (1993). *An introduction to the bootstrap*. New York: Chapman & Hall.

FREDRICKSON, W. E. (1995). A comparison of perceived musical tension and aesthetic response. *Psychology of Music, 23*, 81-87.

FREDRICKSON, W. E. (1997). Elementary, middle, and high school student perceptions of tension in music. *Journal of Research in Music Education, 45*, 626-635.

FREDRICKSON, W. E. (1999). Effect of musical performance on perception of tension in Gustav Holst's First Suite in E-flat. *Journal of Research in Music Education, 47*, 44-52.

FREDRICKSON, W. E. (2000). Perception of tension in music: Musicians versus nonmusicians. *Journal of Music Therapy, 37*, 40-50.

FRIBERG, A., & SUNDBERG, J. (1999). Does music performance allude to locomotion? A model of final ritardandi derived from measurements of stopping runners. *Journal of the Acoustical Society of America, 105*, 1469-1484.

FRIEDLAND, S. (n.d.). Stravinsky's three pieces for clarinet solo. Retrieved June 20, 2002, from http://clarinet.cc/Repertoire003.html

GRICE, H. P. (1975). Logic and conversation. In P. Cole (Ed.), *Syntax and semantics* (Vol. 3, pp. 41-58). New York: Academic Press.

JONES, M. R. (1993). Dynamics of musical patterns: How do melody and rhythm fit together? In T. J. Tighe & W. J. Dowling (Eds.), *Psychology and music: The understanding of melody and rhythm* (pp. 67-92). Hillsdale, NJ: L. Erlbaum.

JUSLIN, P. N., & SLOBODA, J. A. (2001). *Music and emotion: Theory and research*. New York: Oxford University Press.

KRUMHANSL, C. L. (1996). A perceptual analysis of Mozart's Piano Sonata K. 282: Segmentation, tension, and musical ideas. *Music Perception, 13*, 401-432.

KRUMHANSL, C. L. (1997). An exploratory study of musical emotions and psychophysiology. *Canadian Journal of Experimental Psychology, 51*, 336-352.

KRUMHANSL, C. L. (2002). Music: A link between cognition and emotion. *Current Directions in Psychological Science, 11*, 45-50.

KRUMHANSL, C. L., & SCHENCK, D. L. (1997). Can dance reflect the structural and expressive qualities of music? A perceptual experiment on Balanchine's choreography of Mozart's Divertimento No. 15. *Musicae Scientiae, 1* (Spring), 63-85.

KUWANO, S., & NAMBA, S. (1985). Continuous judgment of level-fluctuating sounds and the relationship between overall loudness and instantaneous loudness. *Psychological Research, 47*, 27-37.

LARSON, S. (2004). Musical forces and melodic expectations: Comparing computer models and experimental results. *Music Perception, 21*, 457-498.

LEIBNITZ, G. W. (1686). Nova methodus pro maximis et minimis, itemque tangentibus, qua nec irrationals quantitates moratur [A new method for maxima and minima, and also for tangents, which is not obstructed by irrational qualities]. *Acta Eruditorum*.

LERDAHL, F. (1996). Calculating tonal tension. *Music Perception, 13*, 319-363.

LERDAHL, F. (2001). *Tonal pitch space*. Oxford: Oxford University Press.

LERDAHL, F., & JACKENDOFF, R. (1983). *A generative theory of tonal music*. Cambridge, MA: MIT Press.

LERDAHL, F., & KRUMHANSL, C. L. (2003). La teoría de la tensión tonal y sus consecuencias para la investigación musical [The theory of tonal tension and its implications for musical research]. In J. Martín Galán & C. Villar-Taboada (Eds.), *Los últimos diez años en la investigación musical* (pp. 63-89). Valladolid: Servicio de Publicaciones de la Universidad de Valladolid.

LEVITIN, D. J., NUZZO, R., VINES, B. W., & RAMSAY, J. O. (2005). *Introduction to functional data analysis*. Manuscript submitted for publication.

MADSEN, C. K., & FREDRICKSON, W. E. (1993). The experience of musical tension: A replication of Nielsen's research using the Continuous Response Digital Interface. *Journal of Music Therapy, 30*, 46-63.

MEYER, L. B. (1956). *Emotion and meaning in music*. Chicago: University of Chicago Press.

MCADAMS, S., VINES, B. W., VIEILLARD, S., SMITH, B. K., & REYNOLDS, R. (2004). Influences of large-scale form on continuous ratings in response to a contemporary piece in a live concert setting. *Music Perception, 22*, 297-350.

NEWTON, S. I. (1726/1972). *Philosophial Naturalis Principia Mathematica*. Cambridge, MA: Harvard University Press.

PATEL, A. D. (2003). Language, music, syntax and the brain. *Nature Neuroscience, 6*, 674-681.

PLANTINGA, J., & TRAINOR, L. J. (in press). Memory for melody: Infants use a relative pitch code. *Cognition*.

RAMSAY, J. O. (2000). Functional components of variation in handwriting. *Journal of the American Statistical Association, 95*, 9-15.

RAMSAY, J. O. (2003). *FDA Analysis Toolkit*. Unpublished software application for Matlab, available for download from ftp site ego.psych.mcgill.ca/pub/ramsay/

RAMSAY, J. O., & SILVERMAN, B. W. (1997). *Functional data analysis*. New York: Springer-Verlag.

Ramsay, J. O., & Silverman, B. W. (2002). *Applied functional data analysis: Methods and case studies*. New York: Springer-Verlag.

Repp, B. H. (1992). Diversity and commonality in music performance: An analysis of timing microstructure in Schumann's "Traumerei." *Journal of the Acoustical Society of America, 92*, 2546-2568.

Repp, B. H. (1996). Patterns of note onset asynchronies in expressive piano performance. *Journal of the Acoustical Society of America, 100*, 3917-3932.

Repp, B. H. (1997). Expressive timing in a Debussy prelude: A comparison of student and expert pianists. *Musicae Scientiae, 1*, 257-268.

Russell, J. A. (1979). Affective space is bipolar. *Journal of Personality and Social Psychology, 37*, 345-356.

Saffran, J. R. (2003). Mechanisms of musical memory in infants. In I. Peretz & R. J. Zatorre (Eds.), *The cognitive neuroscience of music* (pp. 32-41). New York: Oxford University Press.

Scherer, K. R., & Zentner, M. R. (2001). Emotional effects of music: Production rules. In P. N. Juslin & J. A. Sloboda (Eds.), *Music and emotion: Theory and research*. Oxford, New York: Oxford University Press.

Scherer, K. R., Zentner, M. R., & Schacht, A. (2002). Emotional states generated by music: An exploratory study of music experts. *Musicae Scientiae*, Special Issue 2001-2002, 149-171.

Schubert, E. (2001). Continuous measurement of self-report emotional response to music. In P. N. Juslin & J. A. Sloboda (Eds.), *Music and emotion: Theory and research* (pp. 393-414). New York: Oxford University Press.

Schubert, E. (2002). Correlation analysis of continuous emotional response to music: Correcting for the effects of serial correlation. *Musicae Scientiae*, Special Issue 2001-2002, 213-236.

Shafie, K., Sigal, B., Siegmund, D., & Worsley, K. J. (2003). Rotation space random fields with an application to fMRI data. *Annals of Statistics, 31*, 1732-1771.

Shepard, R. N. (1984). Ecological constraints on internal representation: Resonant kinematics of perceiving, imagining, thinking, and dreaming. *Psychological Review, 91*, 417-447.

Shepard, R. N. (1987). Toward a universal law of generalization for psychological science. *Science, 237*, 1317-1323.

Shepard, R. N. (1995). Mental universals: Toward a twenty-first century science of mind. In R. L. Solso, & D. W. Massaro (Eds.), *The science of the mind: 2001 and beyond* (pp. 50-62). New York: Oxford University Press.

Sloboda, J. A., & Lehmann, A. C. (2001). Tracking performance correlates of changes in perceived intensity of emotion during different interpretations of a Chopin piano prelude. *Music Perception, 19*, 87-120.

Smith, N. A., & Cuddy, L. L. (2003). Perceptions of musical dimensions in Beethoven's *Waldstein* Sonata: An application of Tonal Pitch Space theory. *Musicae Scientiae*, *7*, 7-34.

Storr, A. (1972). *The dynamics of creation*. New York: Atheneum.

Stravinsky, I. (1920/1993). Second piece for solo clarinet. In *Three pieces*. London: Chester Music Ltd.

Temperley, D. (2001). *The cognition of basic musical structures*. Cambridge, MA: MIT Press.

Todd, N. P. M. (1999). Motion in music: A neurobiological perspective. *Music Perception, 17*, 115-126.

Trehub, S. E. (2003). Musical predispositions in infancy. In I. Peretz & R. J. Zatorre (Eds.), *The cognitive neuroscience of music* (pp. 3-20). New York: Oxford University Press.

Unser, M. (1999). Splines: A perfect fit for signal and image processing. *IEEE Signal Processing Magazine, 16*, 22-38.

Vega, D. A. (2003). A perceptual experiment on harmonic tension and melodic attraction in Lerdahls' Tonal Pitch Space. *Musicae Scientiae, 7*, 35-55.

Vines, B. W., Krumhansl, C. L., Wanderley, M. M., & Levitin, D. J. (in press). Cross-modal interactions in the perception of musical performance. *Cognition*.

Vines, B. W., Wanderley, M. M., Krumhansl, C. L., Nuzzo, R. L., & Levitin, D. J. (2003). Performance gestures of musicians: What structural and emotional information do they convey? In A. Camurri & G. Volpe (Eds.), *Gesture-based communication in human-computer interaction* (pp. 468-478), the proceedings of the 5th International Gesture Workshop, GW 2003, Genova, Italy, April 15-17, 2003. Berlin: Springer Verlag.

Wanderley, M. M. (2002). Quantitative analysis of non-obvious performer gestures. In I. Wachsmuth & T. Sowa (Eds.), *Gesture and sign language in human-computer interaction* (pp. 241-253). Berlin: Springer Verlag.